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Chapter 15

IM DESIGN BELOW 100 KW AND CONSTANT V AND f

15.1. INTRODUCTION

The power of 100 kW is traditionally considered the border between a small and medium power induction machine. In general, sub 100 kW motors use a single stator and rotor stack (no radial cooling channels) and a finned frame washed by air from a ventilator externally mounted at the shaft end ([Figure 15.1](#)). It has an aluminum cast cage rotor and, in general, random wound stator coils made of round magnetic wire with 1 to 6 elementary conductors (diameter $\leq 2.5\text{mm}$) in parallel and 1 to 3 current paths in parallel, depending on the number of pole pairs. The number of pole pairs $2p_1 = 1, 2, 3, \dots 6$.

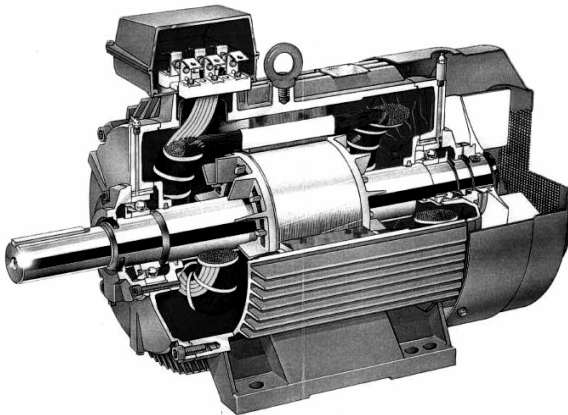


Figure 15.1 Low power 3 phase IM with cage rotor

Induction motors with power below 100 kW constitute a sizable portion of the world electric motor markets. Their design for standard or high efficiency is a nature mixture of art and science, at least in the preoptimization stage. Design optimization will be dealt with separately in a dedicated chapter.

For the most part, IM design methodologies are proprietary.

Here we present what may constitute a sample of such methodologies. For further information, see also [1].

15.2. DESIGN SPECIFICATIONS BY EXAMPLE

Standard design specifications are

- Rated power: $P_n[\text{W}] = 5.5\text{kW}$
- Synchronous speed: $n_1[\text{rpm}] = 1800$
- Line supply voltage: $V_1[\text{V}] = 460\text{V}$
- Supply frequency: $f_1[\text{Hz}] = 60$
- Number of phases $m = 3$
- Phase connections: star
- Targeted power factor: $\cos\phi_n = 0.83$
- Targeted efficiency: $\eta_n = 0.895$ (high efficiency motor)
- p.u. locked rotor torque: $t_{LR} = 1.75$
- p.u. locked rotor current: $i_{LR} = 6$
- p.u. breakdown torque: $t_{bK} = 2.5$
- Insulation class: F; temperature rise: class B
- Protection degree: IP55 – IC411
- Service factor load: 1.0
- Environment conditions: standard (no derating)
- Configuration (vertical or horizontal shaft etc.): horizontal shaft

15.3. THE ALGORITHM

The main steps in IM design are shown in [Figure 15.2](#). The design process may start with (1) design specs and assigned values of flux densities and current densities and (2) calculate in the stator bore diameter D_{is} , stack length, stator slots, and stator outer diameter D_{outs} after stator and rotor currents are found. The rotor slots, back iron height, and cage sizing follows.

All dimensions are adjusted in (3) to standardized values (stator outer diameter, stator winding wire gauge, etc.). Then in (4), the actual magnetic and electric loadings (current and flux densities) are verified.

If the results on magnetic saturation coefficient $(1 + K_{st})$ of stator and rotor tooth are not equal to assigned values, the design restarts (1) with adjusted values of tooth flux densities until sufficient convergence is obtained in $1 + K_{st}$.

Once this loop is surpassed, stages (5) to (8) are traveled by computing the magnetization current I_0 (5); equivalent circuit parameters are calculated in (6), losses, rated slip S_n , and efficiency are determined in (7) and then power factor, locked rotor current and torque, breakdown torque, and temperature rise are assessed in (8).

In (9) all this performance is checked and if found unsatisfactory, the whole process is restarted in (1) with new values of flux densities and/or current densities and stack aspect ratio $\lambda = L/\tau$ (τ – pole pitch).

The decision in (9) may be made based on an optimization method which might result in going back to (1) or directly to (3) when the chosen construction and geometrical data are altered according to an optimization method (deterministic or evolutionary) as shown in Chapter 18.

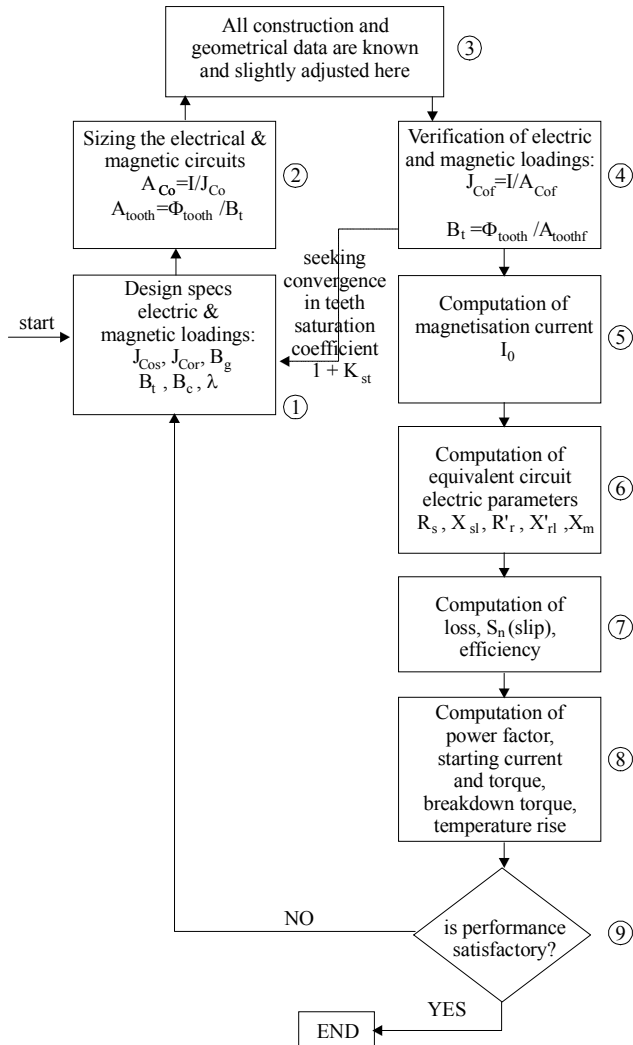


Figure 15.2 The design algorithm

So, IM design is basically an iterative procedure whose output—the resultant machine to be built—depends on the objective function(s) to be minimized and on the corroborating constraints related to temperature rise, starting current (torque), breakdown torque, etc.

The objective function may be active materials or costs or (efficiency)⁻¹ or global costs or a weighted combination of them.

Before treating the optimization stage in Chapter 18, let us perform here a practical design.

15.4. MAIN DIMENSIONS OF STATOR CORE

Here we are going to use the widely accepted $D_{is}^2 L$ output constant concept detailed in the previous chapter. For completely new designs, the rotor tangential stress concept may be used.

Based on this, the stator bore diameter D_{is} (14.15) is

$$D_{is} = \sqrt[3]{\frac{2p}{\pi\lambda} \frac{p_1}{f_1} \frac{S_{gap}}{C_0}}; \quad K_E = 0.98 - 0.005p_1 = 0.97 \quad (15.1)$$

with

$$S_{gap} = \frac{K_E P_n}{\eta_n \cos \phi_{in}}; \quad \lambda = L \left(\frac{2p_1}{\pi D_{is}} \right) = \frac{L}{\tau} \quad (15.2)$$

From past experience, λ is given in [Table 15.1](#).

Table 15.1. Stack aspect ratio λ

$2p_1$	2	4	6	8
λ	0.6 – 1.0	1.2 – 1.8	1.6 – 2.2	2 – 3

From (15.2), the apparent airgap power S_{gap} is

$$S_{gap} = \frac{0.97 \cdot 5.5 \cdot 10^3}{0.895 \cdot 0.83} = 7181.8 \text{ VA}$$

C_0 is extracted from [Figure 14.14](#) for $S_{gap} = 7181.8 \text{ VA}$, $C_0 = 147 \cdot 10^3 \text{ J/m}^3$ and $\lambda = 1.5$, $f_1 = 60 \text{ Hz}$, $p_1 = 2$. So D_{is} from (15.1) is

$$D_{is} = \sqrt[3]{\frac{2 \cdot 2 \cdot 2}{\pi \cdot 1.5 \cdot 60} \cdot \frac{7181.8}{147 \cdot 10^3}} = 0.1116 \text{ m}$$

The stack length L (from 15.2) is

$$L = \frac{1.5\pi \cdot 0.1116}{2 \cdot 2} = 0.1315 \text{ m}$$

The pole pitch

$$\tau = \frac{\pi \cdot 0.1116}{2 \cdot 2} = 0.0876 \text{ m}$$

The number of stator slots per pole $3q$ may be $3 \cdot 2 = 6$ or $3 \cdot 3 = 9$. For $q = 3$, the slot pitch τ_s will be around

$$\tau_s = \frac{\tau}{3q} = \frac{0.0876}{3 \cdot 3} = 9.734 \cdot 10^{-3} \text{ m} \quad (15.3)$$

In general the larger q gives better performance (space field harmonics and losses are smaller).

The slot width at airgap is to be around 5 to 5.3 mm with a tooth of 4.7 to 4.4 mm which is mechanically feasible.

From past experience (or from optimal lamination concept, developed later in this chapter), the ratio of the internal to external stator diameter D_{is}/D_{out} , bellow 100 kW for standard motors is given in [Table 15.2](#).

Table 15.2. Inner/outer stator diameter ratio

$2p_1$	2	4	6	8
$\frac{D_{is}}{D_{out}}$	0.54 – 0.58	0.61 – 0.63	0.68 – 0.71	0.72 – 0.74

With $2p_1 = 4$, we choose $\frac{D_{is}}{D_{out}} = K_D = 0.62$ and thus

$$D_{out} = \frac{D_{is}}{K_D} = \frac{0.1116}{0.62} = 0.18\text{m} \quad (15.4)$$

Let us suppose that this value is normalized. The airgap value has also been introduced in Chapter 14 as

$$\begin{aligned} g &= (0.1 + 0.02 \cdot \sqrt[3]{P_n}) \cdot 10^{-3} \text{ m} \quad \text{for } 2p_1 = 2 \\ g &= (0.1 + 0.012 \cdot \sqrt[3]{P_n}) \cdot 10^{-3} \text{ m} \quad \text{for } 2p_1 \geq 2 \end{aligned} \quad (15.5)$$

In our case,

$$g = (0.1 + 0.012 \cdot \sqrt[3]{5500}) \cdot 10^{-3} = 0.3111 \cdot 10^{-3} \approx 0.35 \cdot 10^{-3} \text{ m}$$

As known, too small airgap would produces large space airgap field harmonics and additional losses while a too large one would reduce the power factor and efficiency.

15.5. THE STATOR WINDING

Induction motor windings have been presented in Chapter 4. Based on such knowledge, we choose the number of stator slots N_s .

$$N_s = 2p_1 q m = 2 \cdot 2 \cdot 3 \cdot 3 = 36 \quad (15.6)$$

A two layer winding with chorded coils: $y/\tau = 7/9$ is chosen as $7/9 = 0.777$ is close to 0.8, which would reduce the first (5^{th} order) stator mmf space harmonic.

The electrical angle between emfs in neighboring slots α_{ec} is

$$\alpha_{ec} = \frac{2\pi p_1}{N_s} = \frac{2\pi 2}{36} = \frac{\pi}{9} \quad (15.7)$$

The largest common divisor of N_s and p_1 ($36, 2$) is $t = p_1 = 2$ and thus the number of distinct stator slot emfs $N_s/t = 36/2 = 18$. The star of emf phasors has 18 arrows (Figure 15.3a) and the distribution of phases in slots of Figure 15.3b.

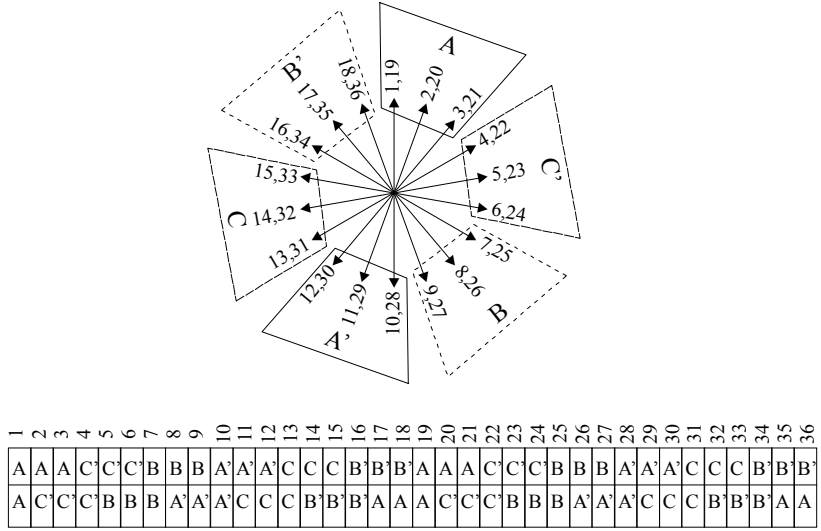


Figure 15.3. A 36 slots, $2p_1 = 4$ poles, 2 layer, chorded coils ($y/\tau = 7/9$) three phase winding

The zone factor K_{q1} is

$$K_{q1} = \frac{\sin \frac{\pi}{6}}{q \sin \left(\frac{\pi}{6q} \right)} = \frac{0.5}{3 \sin \left(\frac{\pi}{18} \right)} = 0.9598 \quad (15.8)$$

The chording factor K_{y1} is

$$K_{y1} = \sin \frac{\pi y}{2 \tau} = \sin \frac{\pi 7}{2 9} = 0.9397 \quad (15.9)$$

So, the stator winding factor K_{w1} becomes

$$K_{w1} = K_{q1} K_{y1} = 0.9598 \cdot 0.9397 = 0.9019$$

The number of turns per phase is based on the pole flux ϕ ,

$$\phi = \alpha_i \tau L B_g \quad (15.10)$$

The airgap flux density is recommended in the intervals

$$\begin{aligned}
B_g &= (0.5 - 0.75)T \quad \text{for } 2p_1 = 2 \\
B_g &= (0.65 - 0.78)T \quad \text{for } 2p_1 = 4 \\
B_g &= (0.7 - 0.82)T \quad \text{for } 2p_1 = 6 \\
B_g &= (0.75 - 0.85)T \quad \text{for } 2p_1 = 8
\end{aligned} \tag{15.11}$$

The pole spanning coefficient α_i (Chapter 14, [Figure 14.3](#)) depends on the tooth saturation factor $1 + K_{st}$.

Let us consider $1 + K_{st} = 1.4$ with $\alpha_i = 0.729$, $K_f = 1.085$. Now from (15.10) with $B_g = 0.7T$:

$$\phi = 0.729 \cdot 0.0876 \cdot 0.1315 \cdot 0.7 = 5.878 \cdot 10^{-3} \text{ Wb}$$

The number of turns per phase W_1 (from Chapter 14, (14.9)) is:

$$W_1 = \frac{K_E V_{lph}}{4K_f K_{wl} f_1 \phi} = \frac{0.97 \cdot \left(\frac{460}{\sqrt{3}} \right)}{4 \cdot 1.085 \cdot 0.902 \cdot 60 \cdot 5.878 \cdot 10^{-3}} = 186.8 \text{ turns/phase} \tag{15.12}$$

The number of conductors per slot n_s is

$$n_s = \frac{a_1 W_1}{p_1 q} \tag{15.13}$$

where a_1 is the number of current paths in parallel.

In our case, $a_1 = 1$ and

$$n_s = \frac{1 \cdot 186.8}{2 \cdot 3} = 31.33 \tag{15.14}$$

It should be an even number as there are two distinct coils per slot in a double layer winding, $n_s = 30$. Consequently, $W_1 = p_1 q n_s = 2 \cdot 3 \cdot 30 = 180$.

Going back to (15.12), we have to recalculate the actual airgap flux density B_g .

$$B_g = 0.7 \cdot \frac{186.8}{180} = 0.726T \tag{15.15}$$

The rated current I_{ln} is

$$I_{ln} = \frac{P_n}{\eta_n \cos \varphi_n \sqrt{3} V_1} = \frac{5500}{0.895 \cdot 0.83 \cdot 1.73 \cdot 460} = 9.303A \tag{15.16}$$

As high efficiency is required and, in general, at this power level and speed, winding losses are predominant from the recommended current densities.

$$\begin{aligned}
J_{\cos} &= (4 \dots 7) A / \text{mm}^2 \quad \text{for } 2p_1 = 2, 4 \\
J_{\cos} &= (5 \dots 8) A / \text{mm}^2 \quad \text{for } 2p_1 = 6, 8
\end{aligned} \tag{15.17}$$

we choose $J_{\cos} = 4.5 \text{ A/mm}^2$.

The magnetic wire cross section A_{Co} is

$$A_{Co} = \frac{I_{ln}}{J_{\cos} a_l} = \frac{9.303}{4.5 \cdot 1} = 2.06733 \text{ mm}^2 \quad (15.18)$$

Table 15.3. Standardized magnetic wire diameter

Rated diameter [mm]	Insulated diameter [mm]
0.3	0.327
0.32	0.348
0.33	0.359
0.35	0.3795
0.38	0.4105
0.40	0.4315
0.42	0.4625
0.45	0.4835
0.48	0.515
0.50	0.536
0.53	0.567
0.55	0.5875
0.58	0.6185
0.60	0.639
0.63	0.6705
0.65	0.691
0.67	0.7145
0.70	0.742
0.71	0.7525
0.75	0.749
0.80	0.8455
0.85	0.897
0.90	0.948
0.95	1.0
1.0	1.051
1.05	1.102
1.10	1.153
1.12	1.173
1.15	1.2035
1.18	1.2345
1.20	1.305
1.25	1.305
1.30	1.356
1.32	1.3765
1.35	1.407
1.40	1.4575
1.45	1.508
1.5	1.559

With the wire gauge diameter d_{Co}

$$d_{Co} = \sqrt{\frac{4A_{Co}}{\pi}} = \sqrt{\frac{4 \cdot 2.06733}{\pi}} = 1.622 \text{ mm} \quad (15.19)$$

In general, if $d_{Co} > 1.3$ mm in low power IMs, we may use a few conductors in parallel a_p .

$$d_{Co}' = \sqrt{\frac{4A_{Co}}{\pi a_p}} = \sqrt{\frac{4 \cdot 2.06733}{\pi \cdot 2}} = 1.15 \text{ mm} \quad (15.20)$$

Now we have to choose a standardized bare wire diameter from [Table 15.3](#).

The value of 1.15 mm is standardized, so each coil is made of 15 turns and each turn contains 2 elementary conductors in parallel (diameter $d_{Co}' = 1.15$ mm).

If the number of conductors in parallel $a_p > 4$, the number of current paths in parallel has to be increased. If, even in this case, a solution is not found, use is made of rectangular cross section magnetic wire.

15.6. STATOR SLOT SIZING

As we know by now, the number of turns per slot n_s and the number of conductors in parallel a_p with the wire diameter d_{Co}' , we may calculate the useful slot area A_{su} provided we adopt a slot fill factor K_{fill} . For round wire, $K_{fill} \approx 0.35$ to 0.4 below 10 kW and 0.4 to 0.44 above 10 kW.

$$A_{su} = \frac{\pi d_{Co}'^2 a_p n_s}{4 K_{fill}} = \frac{\pi \cdot 1.15^2 \cdot 2 \cdot 30}{4 \cdot 0.40} = 155.7 \text{ mm}^2 \quad (15.21)$$

For the case in point, trapezoidal or rounded semiclosed shape is recommended ([Figure 15.4](#)).

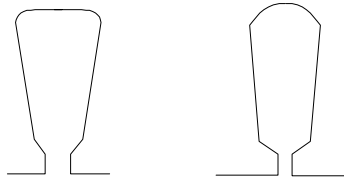


Figure 15.4 Recommended stator slot shapes

For such slot shapes, the stator tooth is rectangular ([Figure 15.5](#)). The variables b_{os} , h_{os} , h_w are assigned values from past experience: $b_{os} = 2$ to 3 mm $\leq 8g$, $h_{os} = (0.5 \text{ to } 1.0)$ mm, wedge height $h_w = 1$ to 4 mm.

The stator slot pitch τ_s (from 15.3) is $\tau_s = 9.734$ mm.

Assuming that all the airgap flux passes through the stator teeth:

$$B_g \tau_s L \approx B_{ts} b_{ts} L K_{Fe} \quad (15.22)$$

$K_{Fe} \approx 0.96$ for 0.5 mm thick lamination constitutes the influence of lamination insulation thickness.

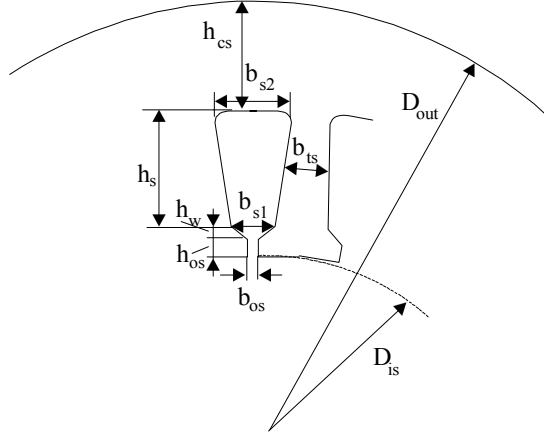


Figure 15.5 Stator slot geometry

With $B_{is} = 1.5 - 1.65 \text{ T}$, ($B_{is} = 1.55 \text{ T}$), from (15.22) the tooth width b_{ts} may be determined.

$$b_{ts} = \frac{0.726 \cdot 9.734 \cdot 10^{-3}}{1.55 \cdot 0.96} = 4.75 \cdot 10^{-3} \text{ m}$$

From technological limitations, the tooth width should not be under $3.5 \cdot 10^{-3} \text{ m}$.

With $b_{os} = 2.2 \cdot 10^{-3} \text{ m}$, $h_{os} = 1 \cdot 10^{-3} \text{ m}$, $h_w = 1.5 \cdot 10^{-3} \text{ m}$, the slot lower width b_{s1} is

$$\begin{aligned} b_{s1} &= \frac{\pi(D_{is} + 2h_{os} + 2h_w)}{N_s} - b_{ts} = \\ &= \frac{\pi(111.6 + 2 \cdot 1 + 2 \cdot 1.5) \cdot 10^{-3}}{36} - 4.75 \cdot 10^{-3} = 5.42 \cdot 10^{-3} \text{ m} \end{aligned} \quad (15.23)$$

The useful area of slot A_{su} may be expressed as:

$$A_{su} = h_s \frac{(b_{s1} + b_{s2})}{2} \quad (15.24)$$

Also,
$$b_{s2} \approx b_{s1} + 2h_s \tan \frac{\pi}{N_s} \quad (15.25)$$

From these two equations, the unknowns b_{s2} and h_s may be found.

$$b_{s2}^2 - b_{s1}^2 = 4A_{su} \tan \frac{\pi}{N_s} \quad (15.26)$$

$$b_{s2} = \sqrt{4A_{su} \tan \frac{\pi}{N_s} + b_{s1}^2} = 10^{-3} \sqrt{4 \cdot 155.72 \tan \frac{\pi}{36} + 5.42^2} \approx 9.16 \cdot 10^{-3} \text{ m} \quad (15.27)$$

The slot useful height h_s (15.24) writes

$$h_s = \frac{2A_{su}}{b_{s1} + b_{s2}} = \frac{2 \cdot 155.72}{5.42 + 9.16} \cdot 10^{-3} = 21.36 \cdot 10^{-3} \text{ m} \quad (15.28)$$

Now we proceed in calculating the teeth saturation factor $1 + K_{st}$ by assuming that stator and rotor tooth produce same effects in this respect.

$$1 + K_{st} = 1 + \frac{F_{mts} + F_{mtr}}{F_{mg}} \quad (15.29)$$

The airgap mmf F_{mg} is

$$F_{mg} \approx 1.2 \cdot g \cdot \frac{B_g}{\mu_0} = 1.2 \cdot 0.35 \cdot 10^{-3} \cdot \frac{0.726}{1.256 \cdot 10^{-6}} = 242.77 \text{ Atturns}$$

with $B_{ts} = 1.55 \text{ T}$, from the magnetization curve table (Table 15.4), $H_{ts} = 1760 \text{ A/m}$. Consequently, the stator tooth mmf F_{mts} is

$$F_{mts} = H_{ts} (h_s + h_{os} + h_w) = 1760 (21.36 + 1 + 1.5) \cdot 10^{-3} = 41.99 \text{ Atturns} \quad (15.30)$$

Table 15.4. Lamination magnetization curve $B_m(H_m)$

B[T]	H[A/m]	B[T]	H[A/m]
0.05	22.8	1.05	237
0.1	35	1.1	273
0.15	45	1.15	310
0.2	49	1.2	356
0.25	57	1.25	417
0.3	65	1.3	482
0.35	70	1.35	585
0.4	76	1.4	760
0.45	83	1.45	1050
0.5	90	1.5	1340
0.55	98	1.55	1760
0.6	106	1.6	2460
0.65	115	1.65	3460
0.7	124	1.7	4800
0.75	135	1.75	6160
0.8	148	1.8	8270
0.85	162	1.85	11170
0.9	177	1.9	15220
0.95	198	1.95	22000
1.0	220	2.0	34000

From (15.29), we may calculate the value of rotor tooth mmf F_{mtr} which corresponds to $1 + K_{st} = 1.4$.

$$F_{mtr} = K_{st} F_{mg} - F_{mts} = 0.4 - \frac{41.99}{242.77} = 55.11 \text{ At/turns} \quad (15.31)$$

As this value is only slightly larger than that of stator tooth, we may go on with the design process.

However, if $F_{mtr} \ll F_{mts}$ (or negative) in (15.31) it would mean that for given $1 + K_{st}$, a smaller value of flux density B_g is required.

Consequently, the whole design procedure has to be brought back to Equation (15.10). The iterative procedure is closed for now when $F_{mtr} \approx F_{mts}$.

As the outer diameter of stator has been calculated in (15.4) at $D_{out} = 0.18 \text{ m}$, the stator back iron height b_{cs} becomes

$$\begin{aligned} b_{cs} &= \frac{D_{out} - (D_{is} + 2(h_{os} + h_w + h_s))}{2} = \\ &= \frac{180 - (111.6 + 2(21.36 + 1.5 + 1))}{2} = 10.34 \text{ mm} \end{aligned} \quad (15.32)$$

The back core flux density B_{cs} has to be verified here with $\phi = 5.878 \cdot 10^{-3} \text{ Wb}$ (from 15.10).

$$B_{cs} = \frac{\phi}{2Lh_{cs}} = \frac{5.878 \cdot 10^{-3}}{2 \cdot 0.1315 \cdot 10.34 \cdot 10^{-3}} = 2.16 \text{ T!!} \quad (15.33)$$

Evidently B_{cs} is too large. There are three main ways to solve this problem. One is to simply increase the stator outer diameter until $B_{cs} \approx 1.4$ to 1.7 T . The second solution consists in going back to the design start (Equation 15.1) and introducing a larger stack aspect ratio λ which eventually would result in a smaller D_{is} , and, finally, a larger back iron height b_{cs} and thus a lower B_{cs} . The third solution is to increase current density and thus reduce slot height h_s . However, if high efficiency is the target, such a solution is to be used cautiously.

Here we decide to modify the stator outer diameter to $D_{out}' = 0.190 \text{ m}$ and thus obtain

$$B_{cs} = 2.16 \frac{b_{cs}}{b_{cs} + \frac{0.190 - 0.180}{2}} = \frac{2.16 \cdot 10.34 \cdot 10^{-3}}{(10.34 + 5) \cdot 10^{-3}} = 1.456 \text{ T}$$

This is considered a reasonable value.

From now on, the outer stator diameter will be $D_{out}' = 0.190 \text{ m}$.

15.7. ROTOR SLOTS

For cage rotors, as shown in Chapters 10 and 11, care must be exercised in choosing the correspondence between the stator and rotor numbers of slots to reduce parasitic torque, additional losses, radial forces, noise, and vibration. Based on past experience (Chapters 10 and 11 backs this up with pertinent

explanations), the most adequate number of stator and rotor slot combinations are given in [Table 15.5](#).

Table 15.5. Stator / rotor slot numbers

$2p_1$	N_s	N_r – skewed rotor slots
2	24	18, 20, 22, 28, 30, ,33,34
	36	25,27,28,29,30,43
	48	30,37,39,40,41
4	24	16,18,20,30,33,34,35,36
	36	28,30,32,34,45,48
	48	36,40,44,57,59
	72	42,48,54,56,60,61,62,68,76
6	36	20,22,28,44,47,49
	54	34,36,38,40,44,46
	72	44,46,50,60,61,62,82,83
8	48	26,30,34,35,36,38,58
	72	42,46,48,50,52,56,60
12	72	69,75,80
	90	86,87,93,94

For our case, let us choose $N_s \neq N_r = 36/28$.

As the starting current is rather large–high efficiency is targeted–the skin effect is not very pronounced. Also, as the locked rotor torque is large, the leakage inductance will not be large. Consequently, from the four typical slot shapes of [Figure 15.6](#), that of [Figure 15.6c](#) is adopted.

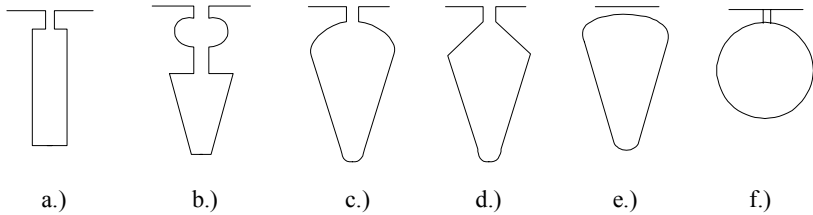


Figure 15.6 Typical rotor cage slots

First, we need the value of rated rotor bar current I_b ,

$$I_b = K_1 \frac{2mW_l K_{wl}}{N_r} I_{ln} \quad (15.34)$$

with $K_1 = 1$, the rotor and stator mmf would have equal magnitudes. In reality, the stator mmf is slightly larger.

$$K_1 \approx 0.8 \cdot \cos \varphi_{ln} + 0.2 = 0.8 \cdot 0.83 + 0.2 = 0.864 \quad (15.35)$$

From (15.34), the bar current I_b is

$$I_b = \frac{0.864 \cdot 2 \cdot 3 \cdot 180 \cdot 0.9019 \cdot 9.303}{28} = 279.6 \text{ A}$$

For high efficiency, the current density in the rotor bar $j_b = 3.42 \text{ A/mm}^2$. The rotor slot area A_b is

$$A_b = \frac{I_b}{j_b} = \frac{279.6}{3.42 \cdot 10^6} = 81.65 \cdot 10^{-6} \text{ m}^2 \quad (15.36)$$

The end ring current I_{er} is

$$I_{er} = \frac{I_b}{2 \sin \frac{\pi p_1}{N_r}} = \frac{279.6}{2 \sin \left(\frac{2\pi}{28} \right)} = 628.255 \text{ A} \quad (15.37)$$

The current density in the end ring $J_{er} = (0.75 - 0.8)j_b$. The higher values correspond to end rings attached to the rotor stack as part of the heat is transferred directly to rotor core.

With $J_{er} = 0.75 \cdot j_b = 0.75 \cdot 3.42 \cdot 10^6 = 2.55 \cdot 10^6 \text{ A/m}^2$, the end ring cross section, A_{er} , is

$$A_{er} = \frac{I_{er}}{J_{er}} = \frac{628.255}{2.565 \cdot 10^6} = 245 \cdot 10^{-6} \text{ m}^2 \quad (15.38)$$

We may now proceed to rotor slot sizing based on the variables defined on [Figure 15.7](#).

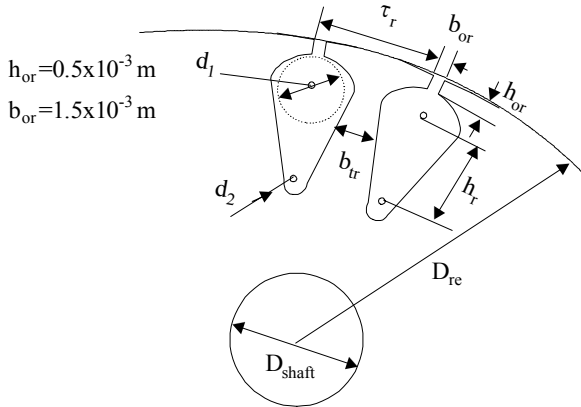


Figure 15.7 Rotor slot geometry

The rotor slot pitch τ_r is

$$\tau_r = \frac{\pi(D_{is} - 2g)}{N_r} = \frac{\pi(111.6 - 0.7) \cdot 10^{-3}}{28} = 12.436 \cdot 10^{-3} \text{ m} \quad (15.39)$$

With the rotor tooth flux density $B_{tr} = 1.60T$, the tooth width b_{tr} is

$$b_{tr} \approx \frac{B_g}{K_{Fe} B_{tr}} \cdot \tau_r = \frac{0.726}{0.96 \cdot 1.6} \cdot 12.436 \cdot 10^{-3} = 5.88 \cdot 10^{-3} m \quad (15.40)$$

The diameter d_1 is obtained from

$$\frac{\pi(D_{re} - 2h_{or} - d_1)}{N_r} = d_1 + b_{tr} \quad (15.41)$$

$$\begin{aligned} d_1 &= \frac{\pi(D_{re} - 2h_{or}) - N_r b_{tr}}{\pi + N_r} = \\ &= \frac{\pi(111.6 - 0.7 - 1) - 28 \cdot 5.88 \cdot 10^{-3}}{\pi + 28} = 5.70 \cdot 10^{-3} m; \quad h_{or} = 0.5 \cdot 10^{-3} m \end{aligned} \quad (15.42)$$

To completely define the rotor slot geometry, we use the slot area equations,

$$A_b = \frac{\pi}{8}(d_1^2 + d_2^2) + \frac{(d_1 + d_2)h_r}{2} \quad (15.43)$$

$$d_1 - d_2 = 2h_r \tan \frac{\pi}{N_r} \quad (15.44)$$

Solving (15.43) and (15.44), we will obtain d_2 and h_r (with $d_1 = 5.70 \cdot 10^{-3} m$, $A_b = 81.65 \cdot 10^{-6} m^2$) as $d_2 = 1.2 \cdot 10^{-3} m$ and $h_r = 20 \cdot 10^{-3} m$.

Now we have to verify the rotor teeth mmf F_{mtr} for $B_{tr} = 1.6T$, $H_{tr} = 2460 A/m$ (Table 15.4).

$$\begin{aligned} F_{mtr} &= H_{tr} \left(h_r + h_{or} + \frac{(d_1 + d_2)}{2} \right) = 2460 \left(20 + 0.5 + \frac{(1.2 + 5.70)}{2} \right) \cdot 10^{-3} \\ &= 60.134 \text{ Atturns} \end{aligned} \quad (15.45)$$

This is rather close to the value of $V_{mtr} = 55.11$ Atturns of (15.39). The design is acceptable so far.

If V_{mtr} had been too large, we might have reduced the flux density, thus increasing tooth width b_{tr} and the bar current density. Increasing the slot height is not practical as already $d_2 = 1.2 \cdot 10^{-3} m$. This bar current density increase could reduce the efficiency below the target value. We may alternatively increase $1 + K_{st}$, and redo the design from (15.10).

When the power factor constraint is not too tight, this is a good solution. To maintain same efficiency, the stator bore diameter has to be increased. So the design should restart from Equation (15.1). The process ends when V_{mtr} is within bounds.

When V_{mtr} is too small, we may increase B_{tr} and return to (15.40) until sufficient convergence is obtained. The required rotor back core may be

calculated after allowing for a given flux density $B_{cr} = 1.4 - 1.7$ T. With $B_{cr} = 1.65$ T, the rotor back core height h_{cr} is

$$h_{cr} = \frac{\phi}{2 \cdot L \cdot B_{cr}} = \frac{5.878 \cdot 10^{-3}}{2 \cdot 0.1315 \cdot 1.65} = 13.55 \cdot 10^{-3} \text{ m} \quad (15.46)$$

The maximum diameter of the shaft D_{shaft} is

$$\begin{aligned} (D_{shaft})_{max} &\leq D_{is} - 2g - 2 \left(h_{or} + \frac{d_1 + d_2}{2} + h_r + h_{cr} \right) = \\ &= \left(111.6 - 2 \cdot 0.35 - 2 \left(1.5 + \frac{(1.2 + 5.69)}{2} + 20 + 13.55 \right) \right) \cdot 10^{-3} \approx 35 \cdot 10^{-3} \text{ m} \end{aligned} \quad (15.47)$$

The shaft diameter corresponds to the rated torque and is given in tables based on mechanical design and past experience. The rated torque is approximately

$$T_{en} = \frac{P_n}{2\pi \frac{f_1}{p_1} (1 - S_n)} \approx \frac{5.5 \cdot 10^{-3}}{2\pi \frac{60}{2} (1 - 0.02)} = 33.56 \text{ Nm} \quad (15.48)$$

For the case in point, the $36 \cdot 10^{-3}$ m left for the shaft diameter suffices.

The end ring cross section is shown in [Figure 15.8](#).

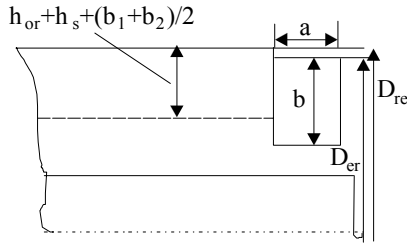


Figure 15.8 End ring cross - section

In general, $D_{re} - D_{cr} = (3 - 4) \cdot 10^{-3}$ m.

Also,

$$b = (1.0 - 1.2) \left(h_r + h_{or} + \frac{(b_1 + b_2)}{2} \right) \quad (15.49)$$

For

$$b = 1.0 \cdot \left(1 + 20 + \frac{(5.69 + 1.2)}{2} \right) = 24.445 \cdot 10^{-3} \text{ m} \quad (15.50)$$

the value of a is

$$a = \frac{A_{er}}{b} = \frac{245 \cdot 10^{-6}}{24.445 \cdot 10^{-3}} = 10.02 \cdot 10^{-3} \text{ m} \quad (15.51)$$

15.8. THE MAGNETIZATION CURRENT

The magnetization mmf F_{1m} is

$$F_{1m} = 2 \left(K_c g \frac{B_g}{\mu_0} + F_{mts} + F_{mtr} + F_{mcs} + F_{mcr} \right) \quad (15.52)$$

So far we have considered $K_c = 1.2$ in F_{mg} (15.29). We do know all of the variables to calculate Carter's coefficient K_c .

$$\gamma_1 = \frac{b_{os}^2}{5g + b_{os}} = \frac{2.2^2 \cdot 10^{-3}}{5 \cdot 0.35 + 2.2} = 1.2253 \cdot 10^{-3} \text{ m} \quad (15.53)$$

$$\gamma_2 = \frac{b_{or}^2}{5g + b_{or}} = \frac{1.5^2 \cdot 10^{-3}}{5 \cdot 0.35 + 1.5} = 0.692 \cdot 10^{-3} \text{ m} \quad (15.54)$$

$$K_{c1} = \frac{\tau_s}{\tau_s - \gamma_1} = \frac{9.734}{9.734 - 1.2253} = 1.144 \quad (15.55)$$

$$K_{c2} = \frac{\tau_r}{\tau_r - \gamma_2} = \frac{12.436}{12.436 - 0.692} = 1.059 \quad (15.56)$$

The total Carter coefficient K_c is

$$K_c = K_{c1} K_{c2} = 1.144 \cdot 1.059 = 1.2115 \quad (15.57)$$

This is very close to the assigned value of 1.2, so it holds. With $F_{mts} = 42$ Atturns (15.30) and $F_{mtr} = 60.134$ Atturns (15.41) as definitive values (for $(1 + K_{st}) = 1.4$), we still have to calculate the back core mmfs F_{mcs} and F_{mcr} .

$$F_{mcs} = C_{cs} \frac{\pi(D_{out} - h_{cs})}{2p_1} H_{cs}(B_{cs}) \quad (15.58)$$

$$F_{mcr} = C_{cr} \frac{\pi(D_{shaft} + h_{cr})}{2p_1} H_{cr}(B_{cr}) \quad (15.59)$$

C_{cs} and C_{cr} are subunitary empirical coefficients that define an average length of flux path in the back core. They may be calculated by AIM methodology, Chapter 5.

$$C_{cs,r} \approx 0.88 \cdot e^{-0.4B_{cs,r}^2} \quad (15.60)$$

with $B_{cs} = 1.456T$ and $B_{cr} = 1.6T$ from Table 15.4 $H_{cs} = 1050$ A/m, $H_{cr} = 2460$ A/m. From (15.58) and (15.59),

$$F_{mcs} = 0.88 \cdot e^{-0.4 \cdot 1.456^2} \frac{\pi(190 - 15.34) \cdot 10^{-3}}{2 \cdot 2} 1050 = 54.22 \text{ At/turns}$$

$$F_{mcr} = 0.88 \cdot e^{-0.4 \cdot 1.6^2} \frac{\pi(36 + 13.55) \cdot 10^{-3}}{2 \cdot 2} 2460 = 23.04 \text{ At/turns}$$

Finally, from (15.52) and (15.29),

$$F_{lm} = 2 \cdot (242.77 + 42 + 60.134 + 54.22 + 23.04) = 844.328$$

The total saturation factor K_s comes from

$$K_s = \frac{F_{lm}}{2F_{mg}} - 1 = \frac{844.328}{2 \cdot 242.77} - 1 = 1.739 \quad (15.61)$$

The magnetization current I_μ is

$$I_\mu = \frac{\pi p_l (F_{lm} / 2)}{3\sqrt{2} W_l K_{wl}} = \frac{\pi \cdot 2 \cdot 844.328}{6\sqrt{2} \cdot 180 \cdot 0.9019} = 3.86 \text{ A} \quad (15.62)$$

The relative (p.u.) value of I_μ is

$$i_\mu = \frac{I_\mu}{I_{ln}} = \frac{3.86}{9.303} = 0.415 = 41.5\% \quad (15.62')$$

15.9. RESISTANCES AND INDUCTANCES

The resistances and inductances refer to the equivalent circuit (Figure 15.9).

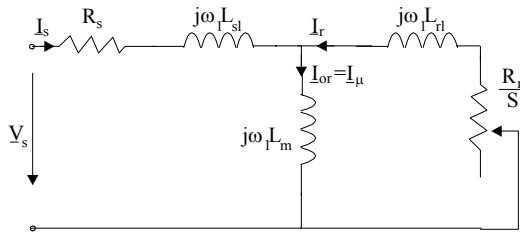


Figure 15.9 The T equivalent circuit (core losses not evident)

The stator phase resistance,

$$R_s = \rho_{Co} \frac{l_c W_l}{A_{co} a_1} \quad (15.63)$$

The coil length l_s includes the active part $2L$ and the end connection part $2l_{end}$

$$l_c = 2(L + l_{end}) \quad (15.64)$$

The end connection length depends on the coil span y , number of poles, shape of coils, and number of layers in the winding.

In general, manufacturing companies developed empirical formulas such as

$$\begin{aligned} l_{end} &= 2y - 0.04 \text{ m for } 2p_1 = 2 \\ l_{end} &= 2y - 0.02 \text{ m for } 2p_1 = 4 \\ l_{end} &= \frac{\pi}{2} y + 0.018 \text{ m for } 2p_1 = 6 \\ l_{end} &= 2.2y - 0.012 \text{ m for } 2p_1 = 8 \end{aligned} \quad (15.65)$$

$\frac{y}{\tau} = \beta$ with β the chording factor. In general,

$$\frac{2}{3} \leq \beta \leq 1 \quad (15.66)$$

In our case for $\frac{y}{\tau} = \frac{7}{9}$, we do have

$$y = \frac{7}{9} \tau = \frac{7}{9} \cdot 0.0876 = 0.06813 \text{ m} \quad (15.67)$$

And from (15.65) for $2p_1 = 4$,

$$l_{end} = 2y - 0.02 = 2 \cdot 0.06813 - 0.02 = 0.11626 \text{ m} \quad (15.68)$$

The copper resistivity at 20°C and 115°C is $(\rho_{Co})_{20^\circ\text{C}} = 1.78 \cdot 10^{-8} \Omega\text{m}$ and $(\rho_{Co})_{115^\circ\text{C}} = 1.37(\rho_{Co})_{20^\circ\text{C}}$. We do not know yet the rated stator temperature but the high efficiency target indicates that the winding temperature should not be too large even if the insulation class is F. We use here $(\rho_{Co})_{80^\circ\text{C}}$.

$$(\rho_{Co})_{80^\circ\text{C}} = (\rho_{Co})_{20^\circ\text{C}} \left(1 + \frac{1}{273} (80 - 20) \right) = 2.1712 \cdot 10^{-8} \Omega\text{m} \quad (15.69)$$

$$\text{From (15.63): } R_s = 2.1712 \cdot 10^{-8} \cdot \frac{2(0.1315 + 0.11626) \cdot 180}{2.06733 \cdot 10^{-6}} = 0.93675 \Omega$$

The rotor bar/end ring segment equivalent resistance R_{be} is

$$R_{be} = \rho_{Al} \left[\frac{L}{A_b} K_R + \frac{l_{er}}{2A_{er} \sin^2 \left(\frac{\pi P_l}{N_r} \right)} \right] \quad (15.70)$$

The cast aluminium resistivity at 20°C $(\rho_{Al})_{20^\circ C} = 3.1 \cdot 10^{-8} \Omega m$ and the end ring segment length l_{er} is

$$l_{er} = \frac{\pi(D_{er} - b)}{N_r} = \frac{\pi(111.6 - 2 \cdot 0.35 - 6 - 24.445) \cdot 10^{-3}}{28} = 9.022 \cdot 10^{-3} m \quad (15.71)$$

K_R , the skin effect resistance coefficient for the bar (Chapter 9, Equation 9.1), is approximately (as for a rectangular bar)

$$K_R = \xi \frac{(\sinh 2\xi + \sin 2\xi)}{(\cosh 2\xi - \cos 2\xi)} \approx \xi \quad (15.72)$$

$$\xi = \beta_s h_r \sqrt{S}; \quad \beta_s = \sqrt{\frac{\omega_l \mu_0}{2\rho_{Al}}} = \sqrt{\frac{2\pi 60 \cdot 1.25 \cdot 10^{-6}}{2 \cdot 3.1 \cdot 10^{-8}}} = 87(m)^{-1} \quad (15.73)$$

For $h_r = 20 \cdot 10^{-3} m$ and $S = 1$, $\xi = 87 \cdot 20 \cdot 10^{-3} \cdot 1 = 1.74$. $K_R \approx 1.74$. From (15.70) the value of R_{be} is

$$\begin{aligned} R_{be}^{S=1} &= 3.1 \cdot 10^{-8} \left(1 + \frac{1}{273} (80 - 20) \right) \left(\frac{0.1315 \cdot 1.73}{81.65 \cdot 10^{-6}} + \frac{9.022 \cdot 10^{-3}}{2 \cdot 245 \cdot 10^{-6} \sin^2 \left(\frac{2\pi}{28} \right)} \right) = \\ &= 1.194 \cdot 10^{-4} \Omega \end{aligned}$$

The rotor cage resistance reduced to the stator R_r' is

$$\begin{aligned} (R_r')_{S=1} &= \frac{4m_l}{N_r} (W_l K_{wl})^2 R_{be80^\circ} = \\ &= \frac{4 \cdot 3}{28} (180 \cdot 0.9019)^2 \cdot 1.194 \cdot 10^{-4} = 1.1295 \Omega \end{aligned} \quad (15.74)$$

The stator phase leakage reactance X_{sl} is

$$X_{sl} = 2\mu_0 \omega_l L \frac{W_l^2}{p_l q} (\lambda_s + \lambda_{ds} + \lambda_{ec}); \quad \beta = \frac{y}{\tau} \quad (15.75)$$

$\lambda_s, \lambda_d, \lambda_{ec}$ are the slot differential and end ring connection coefficients:

$$\lambda_s = \left[\frac{2}{3} \frac{h_s}{(b_{s1} + b_2)} + \frac{2h_w}{(b_{os} + b_{s1})} + \frac{h_{os}}{b_{os}} \right] \left(\frac{1+3\beta}{4} \right) =$$

$$= \left[\frac{2}{3} \frac{21.36}{(5.42 + 9.16)} + \frac{2 \cdot 1.5}{(2.2 + 5.42)} + \frac{1}{2.2} \right] \left(\frac{1+3 \cdot (7/9)}{4} \right) = 1.523 \quad (15.76)$$

An expression of λ_{ds} has been developed in Chapter 9, Equation (9.85). An alternative approximation is given here.

$$\lambda_{ds} \approx \frac{0.9 \tau_s q^2 K_{wl}^2 C_s \gamma_{dc}}{K_c g (1 + K_{st})} \quad (15.77)$$

$$C_s = 1 - 0.033 \frac{b_{os}^2}{g \tau_s}$$

$$\begin{aligned} \gamma_{ds} &= (0.11 \sin \varphi_1 + 0.28) \cdot 10^{-2}; \quad \text{for } q = 8 \\ \gamma_{ds} &= (0.11 \sin \varphi_1 + 0.41) \cdot 10^{-2}; \quad \text{for } q = 6 \\ \gamma_{ds} &= (0.14 \sin \varphi_1 + 0.76) \cdot 10^{-2}; \quad \text{for } q = 4 \\ \gamma_{ds} &= (0.18 \sin \varphi_1 + 1.24) \cdot 10^{-2}; \quad \text{for } q = 3 \\ \gamma_{ds} &= (0.25 \sin \varphi_1 + 2.6) \cdot 10^{-2}; \quad \text{for } q = 2 \\ \gamma_{dc} &= 9.5 \cdot 10^{-2}; \quad \text{for } q = 1 \\ \varphi_1 &= \pi(6\beta - 5.5) \end{aligned} \quad (15.78)$$

For $\beta = 7/9$ and $q = 3$, γ_{ds} (from (15.78)) is

$$\gamma_{ds} = \left[0.18 \sin \left(\pi \left(\frac{6 \cdot 7}{9} - 5.5 \right) \right) + 1.24 \right] \cdot 10^{-2} = 1.15 \cdot 10^{-2}$$

$$C_s = 1 - 0.033 \frac{2.2^2}{0.35 \cdot 9.734} = 0.953$$

From (15.77),

$$\lambda_{ds} = \frac{0.9 \cdot 9.734 \cdot 10^{-3} \cdot 3^2 \cdot 0.9019^2 \cdot 0.953 \cdot 1.15 \cdot 10^{-2}}{1.21 \cdot 0.35 \cdot 10^{-3} (1 + 0.4)} = 1.18$$

For two-layer windings, the end connection specific geometric permeance coefficient λ_{ec} is

$$\lambda_{ec} = 0.34 \frac{q}{L} (l_{end} - 0.64 \cdot \beta \cdot \tau) =$$

$$= 0.34 \cdot \frac{3}{0.1315} \left(0.11626 - 0.64 \cdot \frac{7}{9} \cdot 0.0876 \right) = 0.5274 \quad (15.79)$$

From (15.75) the stator phase reactance X_{sl} is

$$X_{sl} = 2 \cdot 1.126 \cdot 10^{-6} \cdot 2\pi 60 \cdot 0.1315 \cdot \frac{180^2}{2 \cdot 3} (1.523 + 0.18 + 0.5274) = 2.17 \Omega \quad (15.80)$$

The equivalent rotor bar leakage reactance X_{be} is

$$X_{be} = 2\pi f_l \mu_0 L (\lambda_r K_X + \lambda_{dr} + \lambda_{er}) \quad (15.80')$$

where λ_r are the rotor slot, differential, and end ring permeance coefficient. For the rounded slot of [Figure 15.7](#) (see Chapter 6),

$$\lambda_r \approx 0.66 + \frac{2h_r}{3(d_1 + d_2)} + \frac{h_{or}}{b_{or}} = 0.66 + \frac{2 \cdot 20}{3(5.7 + 1.2)} + \frac{0.5}{1.5} = 2.922 \quad (15.81)$$

The value of λ_{dr} is (Chapter 6)

$$\lambda_{dr} = \frac{0.9 \tau_r \gamma_{dr}}{K_c g} \left(\frac{N_r}{6p_1} \right)^2; \quad \gamma_{dr} = 9 \left(\frac{6p_1}{N_r} \right)^2 10^{-2} \quad (15.82)$$

$$\gamma_{dr} = 9 \left(\frac{6 \cdot 2}{28} \right)^2 10^{-2} = 1.653 \cdot 10^{-2}$$

$$\lambda_{dr} = \frac{0.9 \cdot 12.436}{1.21 \cdot 0.35} \cdot 1.653 \cdot 10^{-2} \left(\frac{28}{6 \cdot 2} \right)^2 = 2.378$$

$$\begin{aligned} \lambda_{er} &= \frac{2.3(D_{er} - b)}{N_r \cdot L \cdot 4 \sin^2 \left(\frac{\pi p_1}{N_r} \right)} \log \frac{4 \cdot 7(D_{er} - b)}{b + 2a} = \\ &= \frac{2.3 \cdot 80.455}{28 \cdot 131.5 \cdot 4 \sin^2 \left(\frac{\pi \cdot 2}{28} \right)} \log \left(\frac{4 \cdot 7 \cdot 80.455}{24.445 + 20} \right) = 0.2255 \end{aligned} \quad (15.83)$$

The skin effect coefficient for the leakage reactance K_x is, for $\xi = 1.74$,

$$K_x \approx \frac{3}{2\xi} \frac{(\sinh(2\xi) - \sin(2\xi))}{(\cosh(2\xi) - \cos(2\xi))} \approx \frac{3}{2\xi} = 0.862 \quad (15.84)$$

From (15.80), X_{be} is

$$X_{be} = 2\pi 60 \cdot 1.256 \cdot 10^{-6} \cdot 0.1315 (2.922 \cdot 0.862 + 2.378 + 0.2255) = 3.1877 \cdot 10^{-4} \Omega$$

The rotor leakage reactance X_{rl} becomes

$$X_{rl} = 4m \frac{(W_l K_{wl})^2}{N_r} X_{be} = 12 \cdot \frac{(180 \cdot 0.9019)^2}{28} 3.1387 \cdot 10^{-4} = 3.6506 \Omega \quad (15.85)$$

For zero speed ($S = 1$), both stator and rotor leakage reactances are reduced due to leakage flux path saturation. This aspect has been treated in detail in Chapter 9. For the power levels of interest here, with semiclosed stator and rotor slots:

$$\begin{aligned} (X_{sl})_{sat}^{S=1} &= X_{sl} (0.7 - 0.8) \approx 2.17 \cdot 0.75 = 1.625 \Omega \\ (X_{rl})_{sat}^{S=1} &= X_{rl} (0.6 - 0.7) \approx 3.938 \cdot 0.65 = 2.56 \Omega \end{aligned} \quad (15.86)$$

For rated slip (speed), both skin and leakage saturation effects have to be eliminated ($K_R = K_x = 1$).

From (15.70), R_{be80^0} is

$$\begin{aligned} (R_{be80^0})_{S_n} &= 3.1 \cdot 10^{-8} \left(1 + \frac{1}{273} (80 - 20) \right) \left[\frac{0.1315 \cdot 1}{81.65 \cdot 10^{-6}} + \frac{9.022 \cdot 10^{-3}}{2.245 \cdot 10^{-6} \sin^2 \left(\frac{\pi \cdot 2}{28} \right)} \right] = \\ &= 0.7495 \cdot 10^{-3} \Omega \end{aligned}$$

So the rotor resistance $(R_r')_{S_n}$ is

$$(R_r')_{S_n} = (R_r')_{S=1} \cdot \frac{R_{be80^0}^{S=S_n}}{R_{be80^0}^{S=1}} = 1.1295 \cdot \frac{0.7495 \cdot 10^{-4}}{1.194 \cdot 10^{-4}} = 0.709 \Omega \quad (15.87)$$

In a similar way, the equivalent rotor leakage reactance at rated slip S_n , $X_{rl}^{S=S_n} = 3.938 \Omega$.

The magnetization X_m is

$$X_m = \sqrt{\left(\frac{V_{ph}}{I_\mu} \right)^2 - R_s^2 - X_{sl}} = \sqrt{\left(\frac{460}{3.86\sqrt{3}} \right)^2 - 0.93675^2 - 2.17} = 66.70 \Omega \quad (15.88)$$

Skewing effect on reactances

In general, the rotor slots are skewed. A skewing C of one stator slot pitch τ_s is typical ($c = \tau_s$).

The change in parameters due to skewing is discussed in detail in Chapter 9. Here the approximations are used.

$$X_m = X_m K_{skew} \quad (15.89)$$

$$K_{\text{skew}} = \frac{\sin \frac{\pi c}{2 \tau}}{\frac{\pi c}{2 \tau}} = \frac{\sin \frac{\pi \tau_s}{2 \tau}}{\frac{\pi \tau_s}{2 \tau}} = \frac{\sin \frac{\pi 1}{2 3q}}{\frac{\pi 1}{2 3q}} = \frac{\sin \frac{\pi}{18}}{\frac{\pi}{18}} = 0.9954 \quad (15.90)$$

Now with (15.88) and (15.89),

$$X_m = 66.70 \cdot 0.9954 = 66.3955 \Omega$$

Also, as suggested in Chapter 9, the rotor leakage inductance (reactance) is augmented by a new term X'_{rlskew} .

$$X'_{\text{rlskew}} = X_m (1 - K_{\text{skew}}^2) = 66.70 (1 - 0.9954^2) = 0.6055 \Omega \quad (15.91)$$

So, the final values of rotor leakage reactance at $S = 1$ and $S = S_n$, respectively, are

$$(X_{\text{rl}})_{\text{skew}}^{S=1} = (X_{\text{rl}})_{\text{sat}}^{S=1} + X_{\text{rlskew}} = 2.56 + 0.6055 = 3.165 \Omega \quad (15.92)$$

$$(X_{\text{rl}})_{\text{skew}}^{S=S_n} = X_{\text{rl}} + X_{\text{rlskew}} = 3.6506 + 0.6055 = 4.256 \Omega \quad (15.93)$$

15.10. LOSSES AND EFFICIENCY

The efficiency is defined as output per input power:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{in}} + \sum \text{losses}} \quad (15.94)$$

The loss components are

$$\sum \text{losses} = p_{\text{Co}} + p_{\text{Al}} + p_{\text{iron}} + p_{\text{mv}} + p_{\text{stray}} \quad (15.95)$$

p_{Co} represents the stator winding losses,

$$p_{\text{Co}} = 3 R_s I_{\text{In}}^2 = 3 \cdot 0.93675 \cdot 9.303^2 = 243.215 \text{ W} \quad (15.96)$$

p_{Al} refers to rotor cage losses (at $S = S_n$).

$$p_{\text{Al}} = 3 (R_r)_{S_n} I_m^2 = 3 R_r K_I^2 I_{\text{In}}^2 \quad (15.97)$$

With (15.87) and (15.35), we get

$$p_{\text{Al}} = 3 \cdot 0.709 \cdot 0.864^2 \cdot 9.303^2 = 137.417 \text{ W}$$

The mechanical/ventilation losses are considered as $p_{\text{mv}} = 0.03 P_n$ for $p_1 = 1$, $0.012 P_n$ for $p_1 = 2$, and $0.008 P_n$ for $p_1 = 3, 4$.

The stray losses p_{stray} have been dealt with in detail in Chapter 11. Here their standard value $p_{\text{stray}} = 0.01 P_n$ is considered.

The core loss p_{iron} is made of fundamental p_{iron}^1 and additional (harmonics) p_{iron}^h iron loss.

The fundamental core losses occur only in the teeth and back iron (p_{tl} , p_{yl}) of the stator as the rotor (slip) frequency is low ($f_2 < (3 - 4)\text{Hz}$).

The stator teeth fundamental losses (see Chapter 11) are:

$$p_{\text{tl}} \approx K_t p_{10} \left(\frac{f_1}{50} \right)^{1.3} B_{\text{ts}}^{1.7} G_{\text{tl}} \quad (15.98)$$

where p_{10} is the specific losses in W/Kg at 1.0 Tesla and 50 Hz ($p_{10} = (2 - 3)\text{W/Kg}$; it is a catalog data for the lamination manufacture). $K_t = (1.6 - 1.8)$ accounts for core loss augmentation due to mechanical machining (stamping value depends on the quality of the material, sharpening of the cutting tools, etc.).

G_{tl} is the stator tooth weight,

$$\begin{aligned} G_{\text{tl}} &= \gamma_{\text{iron}} \cdot N_s \cdot b_{\text{ts}} \cdot (h_s + h_w + h_{\text{os}}) \cdot L \cdot K_{\text{Fe}} = \\ &= 7800 \cdot 36 \cdot 4.75 \cdot 10^{-3} \cdot (21.36 + 1.5 + 1) \cdot 10^{-3} \cdot 0.1315 \cdot 0.95 = 3.975 \text{Kg} \end{aligned} \quad (15.99)$$

With $B_{\text{ts}} = 1.55 \text{ T}$ and $f_1 = 60 \text{ Hz}$, from (15.98), p_{tl} is

$$p_{\text{tl}} = 1.7 \cdot 2 \cdot \left(\frac{60}{50} \right)^{1.3} \cdot 1.55^{1.7} \cdot 3.975 = 36.08 \text{W}$$

In a similar way, the stator back iron (yoke) fundamental losses p_{yl} is

$$p_{\text{yl}} = K_y p_{10} \left(\frac{f_1}{50} \right)^{1.3} B_{\text{cs}}^{1.7} G_{\text{yl}} \quad (15.100)$$

Again, $K_y = 1.6 - 1.9$ takes care of the influence of mechanical machining and the yoke weight G_{yl} is

$$\begin{aligned} G_{\text{yl}} &= \gamma_{\text{iron}} \frac{\pi}{4} [D_{\text{out}}^2 - (D_{\text{out}} - 2h_{\text{cs}})^2] \cdot L \cdot K_{\text{Fe}} = \\ &= 7800 \frac{\pi}{4} [0.19^2 - (0.19 - 2 \cdot 15.34 \cdot 10^{-3})^2] \cdot 0.1315 \cdot 0.95 = 8.275 \text{Kg} \end{aligned} \quad (15.101)$$

with $K_y = 1.6$, $B_{\text{cs}} = 1.6 \text{ T}$, p_{yl} from (15.100) is

$$p_{\text{yl}} = 1.6 \cdot 2 \cdot \left(\frac{60}{50} \right)^{1.3} \cdot 1.6^{1.7} \cdot 8.275 = 74.62 \text{W}$$

So, the fundamental iron losses p_{iron}^1 is

$$p_{\text{iron}}^1 = p_{\text{tl}} + p_{\text{yl}} = 36.08 + 74.62 = 110.70 \text{W} \quad (15.102)$$

The tooth flux pulsation core loss constitutes the main components of stray losses (Chapter 11) [1].

$$p_{\text{iron}}^s \approx 0.5 \cdot 10^{-4} \left[\left(N_r \frac{f_l}{p_l} K_{ps} B_{ps} \right)^2 G_{ts} + \left(N_s \frac{f_l}{p_l} K_{pr} B_{pr} \right)^2 G_{tr} \right] \quad (15.103)$$

$$K_{ps} \approx \frac{1}{2.2 - B_{ts}} = \frac{1}{2.2 - 1.55} = 1.5385 \quad (15.104)$$

$$K_{pr} = \frac{1}{2.2 - B_{tr}} = \frac{1}{2.2 - 1.6} = 1.666$$

$$B_{ps} \approx (K_{c2} - 1) B_g = (1.059 - 1.0) \cdot 0.726 = 0.0428 \text{ T} \quad (15.105)$$

$$B_{pr} \approx (K_{c1} - 1) B_g = (1.144 - 1.0) \cdot 0.726 = 0.1045 \text{ T} \quad (15.106)$$

The rotor teeth weight G_{tr} is

$$\begin{aligned} G_{tr} &= \gamma_{\text{iron}} \cdot L \cdot K_{Fe} \cdot N_r \cdot \left(h_r + \frac{d_1 + d_2}{2} \right) \cdot b_{tr} = \\ &= 7800 \cdot 0.1315 \cdot 0.95 \cdot 28 \cdot \left(20 + \frac{5.7 + 1.2}{2} \right) \cdot 10^{-3} \cdot 5.88 \cdot 10^{-3} = 3.710 \text{ Kg} \end{aligned} \quad (15.107)$$

Now, from (15.103),

$$\begin{aligned} p_{\text{iron}}^s &= 0.5 \cdot 10^{-4} \left[\left(28 \cdot \frac{60}{2} \cdot 1.538 \cdot 0.0429 \right)^2 3.975 + \left(36 \cdot \frac{60}{2} \cdot 1.666 \cdot 0.1045 \right)^2 3.710 \right] = \\ &= 7.2 \text{ W} \end{aligned}$$

The total core loss p_{iron} is

$$p_{\text{iron}} = p_{\text{iron}}^l + p_{\text{iron}}^s = 110.70 + 7.2 = 117.90 \text{ W} \quad (15.108)$$

Total losses (15.95) is

$$\sum \text{losses} = 243.215 + 137.417 + 117.90 + 2.2 \cdot 10^{-2} \cdot 5500 = 609.6 \text{ W}$$

The efficiency η_n (from 15.87) becomes

$$\eta_n = \frac{5500}{5500 + 609.6} = 0.9002!$$

The targeted efficiency was 0.895, so the design holds. If the efficiency had been smaller than the target value, the design should have returned to square one

by adopting a larger stator bore diameter D_{is} , then smaller current densities, etc. A larger size machine would have been obtained, in general.

15.11. OPERATION CHARACTERISTICS

The operation characteristics are defined here as active no load current I_{0a} , rated slip S_n , rated torque T_n , breakdown slip and torque S_k , T_{bk} , current I_s and power factor versus slip, starting current, and torque I_{LR} , T_{LR} .

The no load active current is given by the no load losses

$$I_{0a} = \frac{p_{iron} + p_{mv} + 3 \cdot I_{\mu}^2 \cdot R_s}{3V_{ph}} = \frac{117.9 + 56.1 + 3 \cdot 3.86^2 \cdot 0.936}{3(460/\sqrt{3})} = 0.271A \quad (15.109)$$

The rated slip S_n is

$$S_n = \frac{P_{Al}}{P_N + p_{Al} + p_{mv} + p_{stray}} = \frac{137.417}{5500 + 137.417 + 56.1 + 27.5} = 0.024! \quad (15.110)$$

The rated shaft torque T_n is

$$T_n = \frac{P_n}{2\pi \frac{f_l}{p_1} (1 - S_n)} = \frac{5500}{2\pi \frac{60}{2} (1 - 0.024)} = 29.91Nm \quad (15.111)$$

The approximate expressions of torque versus slip (Chapter 7) are

$$T_e = \frac{3p_1}{\omega_l} \frac{V_{ph}^2 \frac{R_r}{S}}{\left(R_s + C_m \frac{R_r}{S}\right)^2 + (X_{sl} + C_m X_{rl})^2} \quad (15.112)$$

with

$$C_m \approx 1 + \frac{X_{sl}}{X_m} = 1 + \frac{2.17}{66.4} = 1.0327 \quad (15.113)$$

From (15.112), the breakdown torque T_{bk} is

$$\begin{aligned} T_{bk} &= \frac{3p_1}{2\omega_l} \frac{V_{ph}^2}{\left[R_s + \sqrt{R_s^2 + (X_{sl} + C_1 X_{rl})^2}\right]} = \\ &= \frac{3 \cdot 2}{2 \cdot 2\pi 60} \frac{(460/\sqrt{3})^2}{\left[0.936 + \sqrt{0.936^2 + (2.17 + 4.256)^2}\right]} = 75.48Nm \end{aligned} \quad (15.114)$$

The starting current I_{LR} is

$$I_{LR} = \frac{V_{ph}}{\sqrt{(R_s + R_r^{S=1})^2 + (X_{sl}^{S=1} + X_{rl}^{S=1})^2}} = \frac{460/\sqrt{3}}{\sqrt{(0.936 + 1.1295)^2 + (1.6275 + 3.165)^2}} = 54.44A \quad (15.115)$$

The starting torque T_{LR} may now be computed as

$$T_{LR} = \frac{3R_r^{S=1}I_{LR}^2}{\omega_1} P_1 = \frac{3 \cdot 1.1295 \cdot 54.44^2 \cdot 2}{2\pi 60} = 53.305Nm \quad (15.116)$$

In the specifications, the rated power factor $\cos\phi_{1n}$, T_{bk}/T_{en} , T_{LR}/T_{en} and I_{LR}/I_{1n} are given. So we have to check the final design values of these constraints.

$$\cos\phi_{1n} = \frac{P_n}{3V_{ph}I_{1n}\eta_n} = \frac{5500}{3 \frac{460}{\sqrt{3}} 9.303 \cdot 0.90} = 0.825 \approx 0.83 \quad (15.117)$$

$$t_{bk} = \frac{T_{bk}}{T_{en}} = \frac{75.48}{29.91} = 2.523 \approx 2.5 \quad (15.118)$$

$$t_{LR} = \frac{T_{LR}}{T_{en}} = \frac{53.305}{29.91} = 1.7828 \approx 1.75 \quad (15.119)$$

$$i_{LR} = \frac{I_{LR}}{I_{1n}} = \frac{54.44}{9.303} = 5.85 \approx 6.0 \quad (15.120)$$

Apparently the design does not need any iterations. This is pure coincidence “combined” with standard specifications. Higher breakdown or starting ratios t_{bk} , t_{LR} would, for example, need lower rotor leakage inductance and higher rotor resistance as influenced by skin effect. A larger stator bore diameter is again required. In general, it is not easy to make a few changes to get the desired operation characteristics. Here the optimization design methods come into play.

15.12. TEMPERATURE RISE

Any electromagnetic design has to be thermally valid (Chapter 12). Only a coarse verification of temperature rise is given here.

First the temperature differential between the conductors in slots and the slot wall $\Delta\theta_{co}$ is calculated.

$$\Delta\theta_{co} \approx \frac{P_{co}}{\alpha_{cond} A_{ls}} \quad (15.121)$$

Then, the frame temperature rise $\Delta\theta_{\text{frame}}$ with respect to ambient air is determined.

$$\Delta\theta_{\text{frame}} = \frac{\sum \text{losses}}{\alpha_{\text{cond}} A_{\text{frame}}} \quad (15.122)$$

For IMs with selfventilators placed outside the motor (below 100kW).

$$\alpha_{\text{conv}} (\text{W} / \text{m}^2 \text{K}) = \begin{cases} 60; & \text{for } 2p_1 = 2; \\ 50; & \text{for } 2p_1 = 4; \\ 40; & \text{for } 2p_1 = 6; \\ 32; & \text{for } 2p_1 = 8; \end{cases} \quad (15.123)$$

More precise values are given in Chapter 12.

The slot insulation conductivity plus its thickness lumped into α_{cond} ,

$$\alpha_{\text{cond}} = \frac{\lambda_{\text{ins}}}{h_{\text{ins}}} = \frac{0.25}{0.3 \cdot 10^{-3}} = 833 \text{ W} / \text{m}^2 \text{K} \quad (15.124)$$

λ_{ins} is the insulation thermal conductivity in (W/m⁰K) and h_{ins} total insulation thickness from the slot middle to teeth wall.

The stator slot lateral area A_{ls} is

$$A_{\text{ls}} \approx (2h_s + b_{s2}) \cdot L \cdot N_s = (2 \cdot 21.36 + 9.16) \cdot 10^{-3} \cdot 0.1315 \cdot 36 = 0.2456 \text{ m}^2 \quad (15.125)$$

The frame area S_{frame} (including the fins area) is

$$A_{\text{frame}} = \pi D_{\text{out}} (L + \tau) \cdot K_{\text{fin}} = \pi \cdot 190 (0.1315 + 0.0876) \cdot 3.0 = 0.392 \text{ m}^2 \quad (15.126)$$

Now, from (15.121)

$$\Delta\theta_{\text{co}} = \frac{234.215}{833 \cdot 0.2456} = 1.18^\circ \text{C}$$

and from (15.122),

$$\Delta\theta_{\text{frame}} = \frac{609.6}{50 \cdot 0.392} = 31.10^\circ \text{C}$$

Suppose that the ambient temperature is $\theta_{\text{amb}} = 40^\circ \text{C}$.

In this case, the winding temperature is

$$\theta_{\text{Co}} = \theta_{\text{amb}} + \Delta\theta_{\text{Co}} + \Delta\theta_{\text{frame}} = 40 + 1.18 + 31.10 = 72.28^\circ \text{C} < 80^\circ \text{C} \quad (15.127)$$

For this particular design, the $K_{\text{fin}} = 3.0$, which represents the frame area multiplication by fins, provided to increase heat transfer, may be somewhat reduced, especially if the ambient temperature would be 20°C as usual.

15.13. SUMMARY

- 100 kW is, in general, considered the border between low and medium power induction machines.
- Low power IMs use a single stator and rotor stack and a finned frame cooled by an external ventilator mounted on the motor shaft.
- Low power IMs use a rotor cage made of cast aluminum and a random wound coil stator winding made of round (in general) magnetic wire with 1 to 6 elementary conductors in parallel and 1 to 3 current paths in parallel.
- The number of poles is $2p_1 = 2, 4, 6, 8, 10$, in general.
- In designing an IM, we understand sizing of the IM; the breakdown torque, starting torque, and current are essential among design specifications.
- The design algorithm contains 9 main stages: design specs, sizing the electric and magnetic circuits, adjustment of sizing data, verification of electric and magnetic loading (with eventual return to step one), computation of magnetization current, of equivalent circuit parameters, of losses, rated slip and efficiency, of power factor, temperature rise, and performance check with eventual returns to step one with adjusted electric and magnetic loadings.
- With values assigned to efficiency and power factor, based on past experience (Esson's output coefficient), the stator bore diameter D_{is} is calculated after assigning a value (dependent on and increasing with the number of poles) for $\lambda = \text{stack length/pole pitch} = 0.6 - 3$.
- The airgap should be small to increase the power factor, but not too small to avoid too high stray losses.
- Double layer, chorded coils stator windings are used in most cases.
- The essence of slot design is the design current density, which depends on the IM design letter A, B, C, D, E, type of cooling, power level, and number of poles. Values in the range of 4 to 8 A/mm² are typical below 100 kW.
- Semiclosed trapezoidal (rounded) stator and rotor slot with rectangular teeth are most adequate below 100 kW.
- A key factor in the design is the teeth saturation factor $1 + K_{st}$ which is assigned an initial value that has to be met after a few design iterations.
- There are special stator/rotor slot number combinations to avoid synchronous parasitic torques, radial forces, noise, and vibration. They are given as [Table 15.5](#).
- An essential design variable is the ratio $K_D = \text{outer/inner stator diameter}$. Its recommended value intervals increase with the number of poles, based on past experience.
- When the lamination cross section is designed to produce the maximum airgap flux density for a given rated current density in the stator [3], the ratio K_D is

$2p_1$	2	4	6	8
K_D	1.58	0.65	0.69	0.72

Using values around these would most probably yield practical designs.

- Care must be exercised to correct the rotor resistance and leakage reactances for skin effect and leakage magnetic saturation at zero speed ($S = 1$). Otherwise, both starting torque and starting current contain notable errors.
- No electromagnetic design is practical unless the temperature rise corresponds to the winding insulation class. Detailed thermal design methodologies are presented in Chapter 12.
- Design optimization methods will be introduced in Chapter 18.

15.14. REFERENCES

1. K. Vogt, Electrical Machines. Design of Rotary Electric Machines, Fourth edition (in German), Chapter 16, VEB Verlag Technik Berlin, 1988.
2. G. Madescu, I. Boldea, T.J.E. Miller, "An Analytical Iterative Model (AIM) for Induction Motor Design", Record of IEEE – IAS – 1996 Annual Meeting, Vol.1, pp.566 – 573.
3. G. Madescu, I. Boldea, T.J.E. Miller, "The Optimal Lamination Approach (OLA) for Induction Motor Design, IEEE Trans Vol.IA – 34, No.2, 1998, pp.1 – 8.